
Three-dimensional Flow about an Arbitrary Blunt Body

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Let us consider an arbitrary blunt body placed in a uniform and steady gas flow having velocity \bar{V}_∞ , pressure p_∞ , density ρ_∞ and Mach number M_∞ . It is well known that in such a case a detached shock wave appears which divides the flow in two parts — the disturbed and non-disturbed one. The flow directly behind a wave will be subsonic at the points where the angle between the velocity vector \bar{V}_∞ and the shock surface is nearly ninety degrees. If a body is bounded in a direction perpendicular to the velocity vector \bar{V}_∞ then somewhere downstream, the flow again becomes supersonic and the sonic surface encloses the subsonic nose region.

Bearing in mind this consideration, let us formulate the problem. For a prescribed body we seek the solution of the equations of gas dynamics in a region situated between the shock wave and the body. The boundary conditions are the usual conditions on the body surface and on the shock the position of which must be determined. If the body has finite dimensions, there is no solution of such a form in the whole neighbourhood of the body. As a rule somewhere downstream the second tail shock wave arises. However this does not affect the supersonic and subsonic regions upstream where the solution may be found independently. So the complete problem is divided into two problems, the subsonic and supersonic ones (Fig. 1). The first problem is to determine the flow near the nose in region I where equations are of a mixed type and the second is the calculation of a flow in purely supersonic region II. The solution of the latter was given earlier in ref. (1).

Here we shall consider the solution of the former problem. The region I has as its boundaries a shock wave, the body surface and a sonic surface, or, more correctly the limiting characteristic surface. The limiting characteristic surface can be defined as a characteristic surface which is farthest away from the sonic surface among those which have common points with the sonic surface. The boundary conditions on the wave and on the body are the usual ones and there is no need for any boundary conditions on the limiting charac-

teristic surface. Now as the subsonic region disturbances are distributed from each point in all directions, the flow should be determined in the whole region I at once. For such problems the so-called stabilisation method has

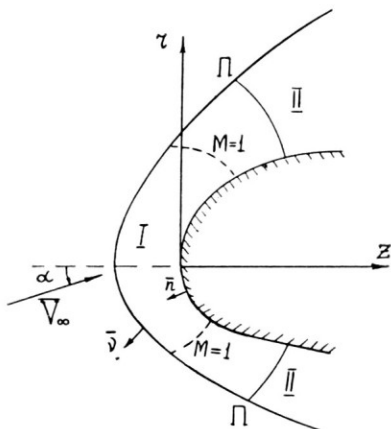


FIG. 1

proved to be very effective. This method is based on the physical fact that a steady flow around a body arises if the body moves for a sufficiently long time at constant velocity. Therefore it should be expected that under constant boundary conditions on the body and at infinity the solution of the non-steady problem will tend to the solution of a steady one as time tends to infinity. Since equations of a non-steady flow are always hyperbolic, the task is reduced to the solution of a mixed problem for a hyperbolic system. The boundary conditions for a given problem should be formulated on the surface of the body and the shock wave, the position of which is not known *a priori*. Instead of the characteristic surface it is reasonable to draw in the supersonic region the surface Π which has a spatial type⁽⁴⁾. As on the characteristic surface it is unnecessary to formulate any boundary conditions on it.

Thus to determine a three-dimensional flow around a blunt body one needs, as a rule, to solve complete equations of gas dynamics. However, the situation may be simplified if the body nose part has a spherical shape and the flow is axisymmetric. It is sufficient to have axial symmetry only in region I with respect to the incident flow velocity vector. It will take place if the intersection of the limit characteristic surface with the body surface lies entirely on its spherical part. Thus, it is possible to compute a three-dimensional flow around spherically blunt cones to considerable angles of attack solving an axisymmetric problem in the nose part^(2, 3).

If the nose part of the body is not spherical it is necessary to solve the

general gas dynamic equations with four independent variables.

$$\left. \begin{aligned} \frac{d\bar{V}}{dt} + \frac{1}{\rho} \text{grad } p = 0; \quad \frac{d}{dt} = \frac{\partial}{\partial t} + \bar{V} \cdot \text{grad} \\ \frac{d\rho}{dt} + \rho \text{div } \bar{V} = 0; \quad c^2 = \left(\frac{\partial p}{\partial \rho} \right)_{S = \text{const}} \\ \frac{dp}{dt} + \rho c^2 \text{div } \bar{V} = 0; \end{aligned} \right\} \quad (1)$$

Boundary conditions are:

(1) On the body:

$$\bar{V}_n = 0 \quad (2)$$

(2) On the shock:

$$\left. \begin{aligned} \rho \bar{V}_v = \rho_\infty \bar{V}_{v\infty} \\ \bar{V} \times \bar{v} = \bar{V}_\infty \times \bar{v} \\ p + \rho \bar{V}_v^2 = p_\infty + \rho_\infty \bar{V}_{v\infty}^2 \\ h + \frac{V_v^2}{2} = h_\infty + \frac{V_{v\infty}^2}{2} \end{aligned} \right\} \quad (3)$$

The speed of sound c and the enthalpy h entering the equations are thermodynamic functions of pressure and density. Their precise form depends on the equations of the state of the gas.

Now let us introduce a co-ordinate system (ξ, η, θ, t) connected with the body and the shock in such a way that the body surface should have an equation $\xi=0$ and the shock surface an equation $\xi=1$. Let us draw in the meridional plane $\theta=\text{const}$ a ray $\Lambda(\eta, \theta)$ from the point $O(z=z'(\eta), r=0)$ at an angle η to the axis z . Let $G(\eta, \theta)=OA$ and $F(\eta, \theta)=OB$ where A, B are points of intersection of the ray Λ with surfaces of the body and the shock wave, respectively (Fig. 2).

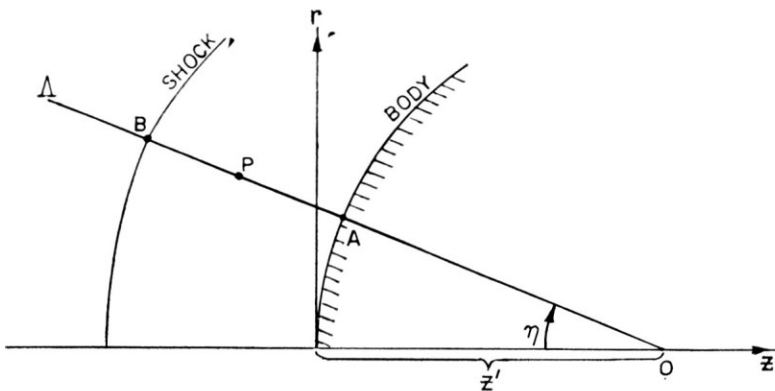


FIG. 2

For each point P of AB put

$$\xi = \frac{AP}{AB} = \frac{R-G}{F-G}$$

The relationship between cylindrical co-ordinates (z, r, ϕ) and (ξ, η, θ) is given by

$$\begin{aligned} z &= z' - R \cos \eta; & r &= R \sin \eta; & \phi &= \theta; \\ R &= G + \xi(F-G) \end{aligned}$$

In the new variables equation (1) becomes

$$\frac{\partial \mathbf{X}}{\partial t} + A \frac{\partial \mathbf{X}}{\partial \xi} + B \frac{\partial \mathbf{X}}{\partial \eta} + C \frac{\partial \mathbf{X}}{\partial \theta} + \Gamma = 0 \quad (1')$$

where

$$\mathbf{X} = \begin{Bmatrix} u \\ v \\ w \\ p \\ \rho \end{Bmatrix}; \quad \Gamma = r^{-1} \begin{Bmatrix} 0 \\ -w^2 \\ vw \\ \rho c^2 v \\ \rho v \end{Bmatrix}$$

Here (u, v, w) are velocity components in co-ordinates (z, r, ϕ) and A, B, C are matrices which depend on the vector \mathbf{X} and the new independent variables.

To write the relation on the wave in new co-ordinates it is sufficient to find expressions for \bar{v} and D . Since the equation of the shock in co-ordinates (z, r, ϕ, τ) is $\xi(z, r, \phi, \tau) = 0$

$$\begin{aligned} \bar{v} &= \frac{\{\xi_z, \xi_r, (1/r)\xi_\phi\}}{\sqrt{[\xi_z^2 + \xi_r^2 + (1/r^2)\xi_\phi^2]}} \\ D &= \frac{\xi_\tau}{\sqrt{[\xi_z^2 + \xi_r^2 + (1/r^2)\xi_\phi^2]}} \end{aligned}$$

Thus finally the problem of determining the flow in region I prescribed by inequalities

$$0 \leq \xi \leq 1; \quad 0 \leq \eta \leq \eta_0; \quad 0 \leq \theta \leq 2\pi$$

is formulated as follows:

Proceeding from initial values \mathbf{X} and F with $t = t_0$ to find the limit solution of a mixed problem for hyperbolic system (equation 1') with boundary conditions (2) and (3) which will satisfy additional steady-state conditions

$$\frac{\partial \mathbf{X}}{\partial t} = 0, \quad \frac{\partial F}{\partial t} = 0$$

On the boundary $\eta = \eta_0$ no conditions have to be put if the hypersurface $\eta = \eta_0$ retains a spatial type.

Let us describe now the difference method for solving the non-stationary problem in region I. It represents a generalisation of the schemes used in the previous works^(1,2,3) for solving problems with three variables. In the region under consideration let us introduce a rectangular net with the steps $\Delta\xi, \Delta\eta, \Delta\theta, \Delta t$ and denote

$$\begin{aligned}\zeta_m &= m \Delta\xi, \quad \eta_k = k \Delta\eta, \quad \theta_l = l \Delta\theta \\ f_{m,k,l}(t) &= f(\zeta_m, \eta_k, \theta_l, t) \\ f_{m+\frac{1}{2},k,l}(t) &= \frac{1}{2}[f_{m,k,l}(t) + f_{m+1,k,l}(t)] \\ f_{m,k,l}(t + \frac{1}{2}\Delta t) &= \frac{1}{2}[f_{m,k,l}(t) + f_{m,k,l}(t + \Delta t)]\end{aligned}$$

Approximate the derivatives as follows:

$$\begin{aligned}\left(\frac{\partial X}{\partial t}\right)_{m+\frac{1}{2},k,l} &= \frac{1}{\Delta t} [X_{m+\frac{1}{2},k,l}(t + \Delta t) - X_{m+\frac{1}{2},k,l}(t)] \\ \left(\frac{\partial X}{\partial \xi}\right)_{m+\frac{1}{2},k,l} &= \frac{1}{\Delta\xi} [X_{m+1,k,l} - X_{m,k,l}]_{t+\frac{1}{2}\Delta t} \\ \left(\frac{\partial X}{\partial \eta}\right)_{m+\frac{1}{2},k,l} &= \frac{1}{2\Delta\eta} [X_{m+\frac{1}{2},k+1,l} - X_{m+\frac{1}{2},k-1,l}]_{t+\frac{1}{2}\Delta t} \\ \left(\frac{\partial X}{\partial \theta}\right)_{m+\frac{1}{2},k,l} &= \frac{1}{2\Delta\theta} [X_{m+\frac{1}{2},k,l+1} - X_{m+\frac{1}{2},k,l-1}]_{t+\frac{1}{2}\Delta t}\end{aligned}$$

The difference equations thus obtained are solved by the double-sweep method in direction ξ and by iteration in directions η and θ . A more detailed description of the method will be published later.

Let us give now some examples of calculations. At the bottom of Fig. 3 the distance between the shock and the sphere in the axisymmetric flow is plotted as a function of Mach number M_∞ for an ideal gas with specific heat ratio 1.4 and for air at altitude 20 and 30 kilometers. The curve for a cylinder in ideal gas is given at the top of the figure.

Figures 4–10 give the results of calculations of the flow round blunt bodies in the absence of symmetry in the subsonic region. Fig. 4 gives the shapes of shock waves for a paraboloid of revolution (on the left) and an elliptical paraboloid with semi-axes 1:0.7746. Figs. 5 and 6 give the distribution of pressure and velocity components on the surface of the paraboloid of revolution for several angles of attack. Figs. 7 and 8 give the same data for an elliptical paraboloid.

To check the method of calculating the three-dimensional flow, a body of a complicated shape with different and sharply varying surface curvature has

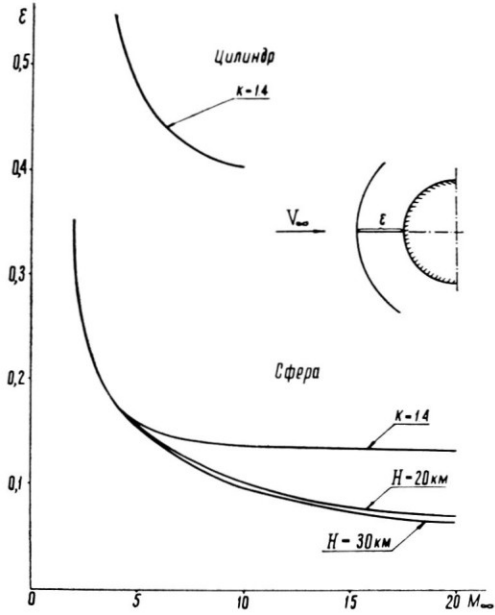


FIG. 3

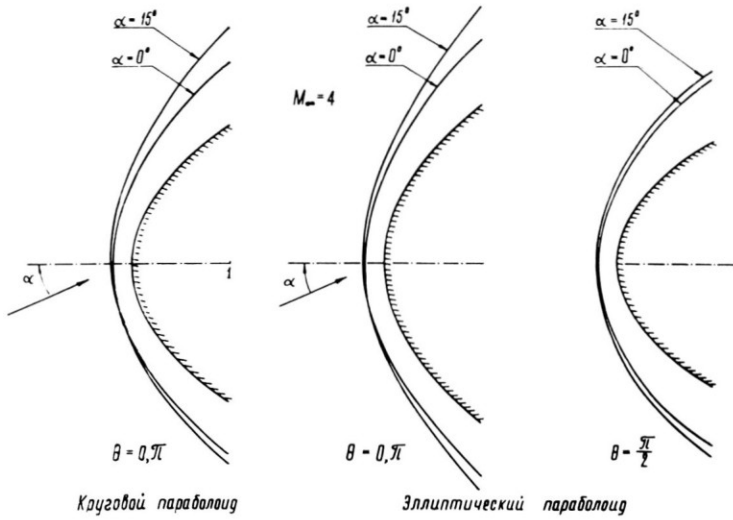


FIG. 4

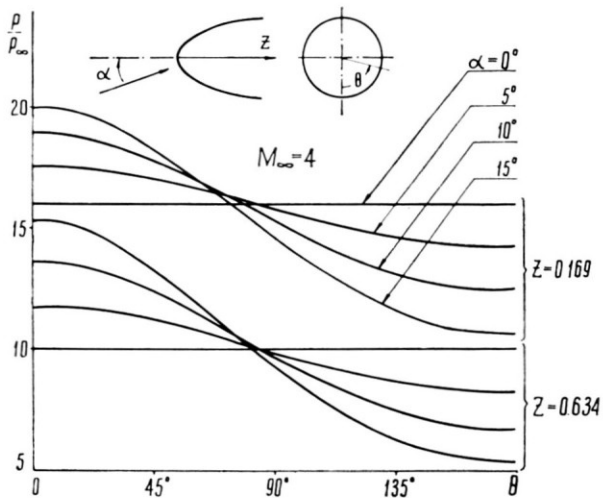


FIG. 5

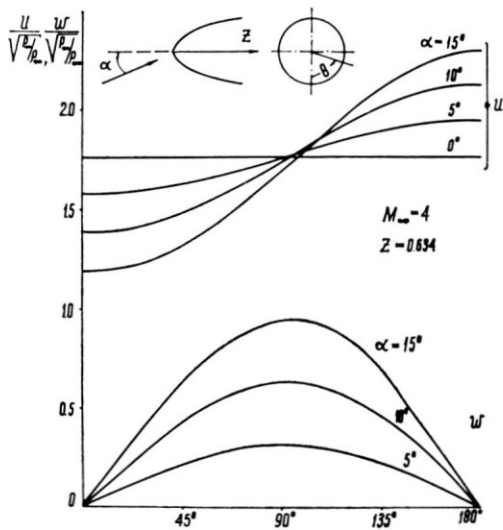


FIG. 6

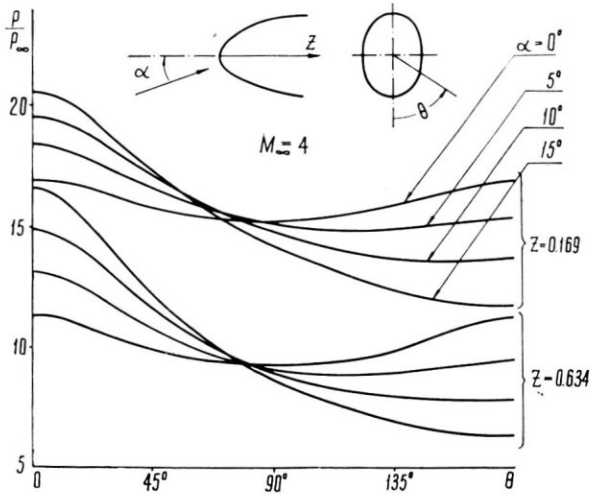


FIG. 7

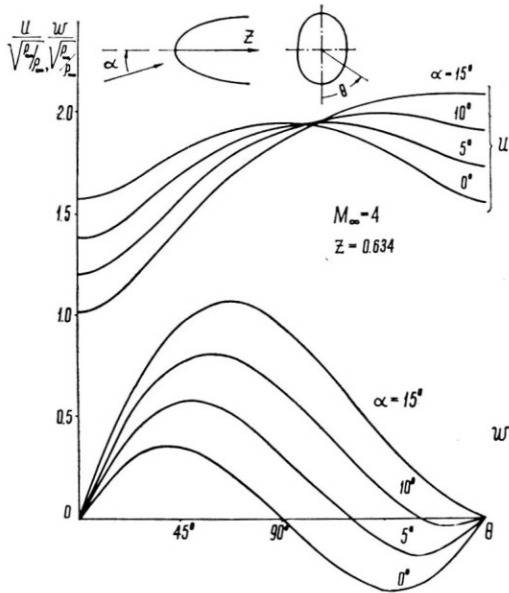


FIG. 8

been specially invented. Its two longitudinal sections and one of the cross-sections are given on Fig. 9. Since the analytical presentation of the surface is

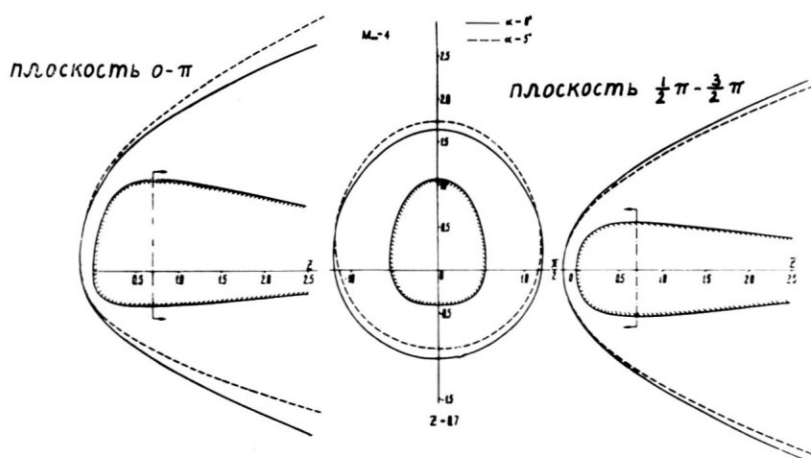


FIG. 9

most convenient for calculations, after the section contours had been drawn they were approximated by functions of a simple form. The shapes of the shock waves around this body are also given for angles of attack 0° and 5° . On Fig. 10 the pressure distribution in several longitudinal sections is plotted as a function of the co-ordinate θ .

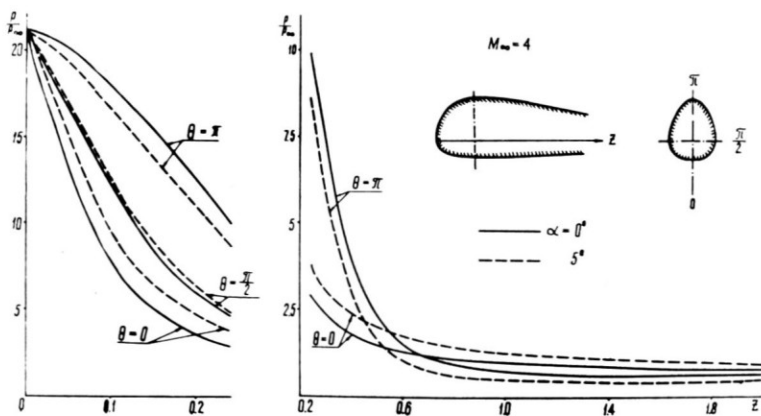


FIG. 10

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DISCUSSION

Dr. K. Kraemer (A.V.A. Bunsenstr. 10, 34 Göttingen, W. Germany): Some of the calculated curves were marked $k=1.4$, others: $H=30$ km etc. Does this mean another value of K (= ratio of specific heats).

Dr. Rusanov: No, another equation of state was used; which accounts for real gas effects assuming equilibrium to be established.

Dr. E. W. E. Rogers (Aero Division, N.P.L., Teddington, England): I was impressed with the variety of hypersonic flow problems that Dr. Rusanov has considered with his theoretical approach. Could he tell us whether the theoretical predictions have been compared with experiment, and how close the agreement was?

Dr. Rusanov: At first it should be noted that the accuracy of our computations has been checked purely mathematically as the accuracy of the numerical solution of the differential equations, without using the experimental data.

However, the comparison with the experiment was made in many cases in order to get information about the validity of the physical model for real conditions. For comparison some carefully made experiments with spheres, cylinders and other bodies were chosen, for example, from the works by R. E. Oliver (*J.A.S.*, 23, February 1956), J. M. Kendall (*Jet Propulsion Lab Progr Rept* 1959) and others. In all the cases the discrepancies between the experiment and numerical solution were in the limits of the accuracy of the experimental data.

Prof. W. J. Prosnak (Inst. of Fundamental Technological Problems, Polish Academy of Sciences, 21 Swietokrzyska Str., Warsaw, Poland): I wish to extend my sincere congratulations to Dr. Rusanov for his spectacular results.

concerning such difficult and fundamental a problem like computing hypersonic flow around an arbitrary three-dimensional blunt body. However, I am interested not only in the results themselves but also in the method and in the cost of obtaining them. Therefore I would like to pose two questions.

First I would like to know if any comparison with other existing methods has been made, e.g. with the method of integral relations of Dorodnicyn. I mean here the comparison concerning the results as well as the computer time needed.

Second, I wonder if Dr. Rusanov could tell us more about the finite difference scheme he used: its order, its stability, its convergence.

I would also like to ask if Dr. Rusanov computed the entropy field as well, and what conclusions has he eventually drawn as far as the position of the maximum entropy line is concerned.

Dr. Rusanov: At a time when our paper was prepared there were no published works known to us on the calculation of three-dimensional subsonic flow around a blunt body. So we could not compare our three-dimensional calculations with the results obtained by other methods. For the axisymmetrical and plane flows the comparisons were made with the results of S. M. Belotzerkovsky for spheres and cylinders obtained by a method of integral relations.

The agreement is good for spheres in the subsonic region and for not too small Mach numbers ($M_\infty \geq 4$). In the supersonic region and for the smaller Mach numbers, as well as for cylinders, the agreement is worse. This is due to the fact that in these cases the functions are not so smooth, and in order to achieve a greater accuracy one has to have more mesh points than it is possible in the method of integral relations.

The computer time for the difference method depends on the required accuracy only and for the accuracy obtained by the method of integral relation it is the same.

A more detailed description of the difference scheme will be published in the near future in the U.S.S.R. *J. of Math. Phys. and Computational Mathematics*.

The entropy field was calculated as well, and from these calculations it can be concluded that the entropy on the surface of the body is constant and equal to its maximum value with the accuracy of the computations made.